Chapter 8

TIME INVARIANCE

In Experiments 2 and 3 several field measurements were made on a daily basis for several irrigation cycles. For example, Figures 3-14, -16 and -18 gave a qualitative impression of the evolution over time of profile water contents for Experiment 2. Other parameters measured included catch can depths, change in storage due to irrigation, field and ML surface temperatures, and evaporation. This chapter will consider whether or not these parameters were time invariant.

Time invariance refers to the tendency of values of a parameter, measured at various locations, to retain their relative ranking over time. For example, profile water contents measured at 5 locations in a field, designated A through E, might be ranked quantitatively as D, C, E, A, and B with location D having the highest value and location B having the lowest. If the profile water contents were measured some time later and the quantitative ranking remained the same then the profile water content could be considered time invariant. This is true even though the mean value of profile water content might have decreased between measurements.

The utility of this concept lies in the possibility of

establishing 1) the time invariance of a particular parameter; and, 2) identifying particular locations that represent important statistics of the parameter such as the mean value. Although these two steps require a large number of measurements, future measurements are reduced to those at the locations identified in step two.

Vachaud et al. (1985), using profile water content data, showed that some locations preserved their rank in the cumulative probability function with small variance in time. Particular locations could be associated with the mean value or extreme values of profile water content. These authors used the Spearman rank correlation coefficient, $r_{\rm s}$, to confirm the presence of time stability (time invariance). The value of $r_{\rm s}$ varies from -1 to 1 with values near zero indicating a lack of both correlation and time invariance. The value of $r_{\rm s}$ can be calculated by:

$$r_{s} = \frac{\sum (R_{ij} - \bar{R}_{j}) (R_{if} - \bar{R}_{f})}{\left[\sum (R_{ij} - \bar{R}_{j})^{2} \sum (R_{if} - \bar{R}_{f})^{2}\right]^{1/2}} [8-1]$$

where R_{ij} and R_{if} are the ranks for location i at times j and f, respectively, and \bar{R}_j and \bar{R}_f are the mean ranks at times j and f, respectively. The summations occur for i = 1 to N where N is the number of locations. Average ranks are used in case of ties. The test statistic for significance of the correlation is:

$$t^* = (n - 2)^{1/2} r_s / (1 - r_s^2)^{1/2}$$
 [8-2]

where t^* is assumed to come from a t distribution with n-2 degrees of freedom (SAS Institute Inc. 1987, p. 270).

In order to identify those locations representative of certain statistics of the cumulative probability function, Vachaud et al. (1985) first defined the relative difference, δ_{ij} , for location i and time j as:

$$\delta_{ij} = (S_{ij} - E[S_{ij}]) / E[S_{ij}]$$
 [8-3]

where E[] is the expected value operator and S_{ij} is the profile water content at location i and time j. They calculated δ_{ij} for all times and locations and found the average relative difference over time, $\bar{\delta}_i$, for each location. Plotting of $\bar{\delta}_i$ versus rank, with error bars for the maximum and minimum relative difference at each location, allowed easy identification both of locations which represented the mean and extreme values and of locations which maintained their relative rank with the most precision.

Kachanoski and De Jong (1988) attempted to refine the definition of time invariance by stating that time invariance exists if the relative difference remains constant over time:

$$\delta_{ij} \doteq \delta_{if}$$
 [8-4]

Writing 8-3 for times j and f gives:

$$\delta_{ij} = S_{ij}/E[S_{ij}] - 1$$
 [8-5]

$$\delta_{if} = S_{if}/E[S_{if}] - 1$$
 [8-6]

and substituting 8-5 and 8-6 into 8-4 gives

$$S_{if} \doteq S_{ij}E[S_{if}]/E[S_{ij}]$$
 [8-7]

Equation 8-7 is a linear relationship between the profile water content at different times with slope $E[S_{if}]/E[S_{ij}]$ and intercept of zero. If 8-4 holds then a simple correlation between S_{if} and S_{ij} is a good test for time stability. Furthermore, a correlation between δ_{ij} and δ_{if} should have a slope of 1 and intercept of 0 (Kachanoski and De Jong, 1988).

For their data, Kachanoski and De Jong (1988) found that the mean relative difference remained constant over time but individual values did not remain constant even though the ranking remained almost the same. Thus 8-4 is a stronger criterion for time invariance than is the Spearman rank correlation test suggested by Vachaud et al. (1985). Clearly if 8-4 holds for a data set then the ranking will remain constant over time but time stable ranking does not imply 8-4 since the variance of δ_i may change with time.

The choice of criterion is then a function of the results desired. If only the assurance of time stability of the location representing the mean is desired, then the criterion of constant ranking will suffice. However, if locations

representing other statistics, for example 1 S.D. from the mean, are required then the stricter requirement of 8-4 is necessary.

One other useful result was pointed out by Kachanoski and De Jong (1988). From 8-3 it is evident that δ_{ij} is a linear transformation of S_{ij} . Therefore the spatial autocorrelation of S_{ij} and δ_{ij} should be identical since linear transformation does not affect the autocorrelation. It follows that if δ_{ij} is time invariant for $j=1,\ldots,N$, in the sense of 8-4, then the autocorrelation of S_{ij} will also be time invariant. It also follows that the normalized semivariogram will be time invariant (but not the ordinary semivariogram), a fact which proves useful if it is desired to combine variograms from data sets measured on different days.

In order to investigate the time invariance of variables measured in Experiments 2 and 3, the relative difference was calculated for each location, day and variable. For each location the mean relative difference over a given time period was calculated and plots were made of the mean relative difference versus rank. "Error" bars were plotted showing the maximum and minimum values of relative difference for each location in order to illustrate the dispersion occurring for a given data set.

Linear correlations were performed on the relative differences comparing each day's data to all other days. The

intercept and slope were determined as well as the significance of the correlation (probability that the slope = 0) and the probability that the slope = 1 (in order to test Equation 8-4). The test statistic for significance of correlation (H_0 : b_1 = 0; H_1 : b_1 \neq 0) was:

$$t^* = (b_1 - \beta_1)/s_{b1}$$
 [8-8]

where t* follows the t distribution with n - 2 degrees of freedom, β_1 equals 0, and s_{b1} is the sample standard deviation of b_1 given by Benjamin and Cornell (1970, Eq. 4.3.27). The same test statistic was used when considering the alternate hypotheses; H_o : b_1 = 1; H_1 : $b_1 \neq 1$; except that β_1 equaled 1. If the slope equaled 1 then the stricter condition for time invariance, implied by Equation 8-4, was met.

The ranking test for time invariance was applied by computing Spearman rank-order correlations as discussed above. The Spearman calculations were done using the SAS statistical package and formulas and significance tests are given by the SAS Institute Inc. (1987).

Results, Experiment 2.

<u>Irrigation Related Parameters</u>.

For each location the mean relative difference, δ_{ij} , for catch can depths was plotted versus the rank based on the mean value of δ_{ij} (Figure 8-1). The error bars show the maximum and minimum value of δ_{ij} for each location and the number below the error bar is the location code number. The spatial distribution of applied depths was quite invariant with time. Linear correlations showed that the data were highly correlated across irrigations (Table 8-1). Slopes were less than 1 and only close to 1 in one instance indicating that catch can depths were not time invariant by the stricter criterion of Equation 8-4.

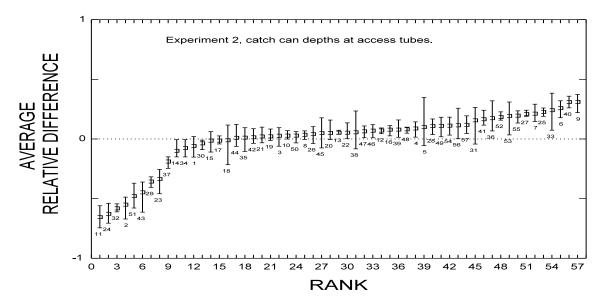


Figure 8-1. Average relative difference for all irrigations of catch can depths at access tubes [squares], ranked and with bars showing maximum and minimum values, Experiment 2.

Table 8-1.

Linear correlations and Spearman rank-order correlations among catch can depths (cm). All irrigations, Experiment 2. For linear correlations, from top to bottom, the numbers are the correlation coefficient, intercept, slope, probability that slope equals 0 (data sets uncorrelated), probability that slope equals 1. For Spearman correlations the numbers are Spearman's coefficient, r, and probability that ranks are not preserved.

	<u>Irrigat</u> :	ion\Irrigation	2	3
Simple	1	r	0.8570	0.8275
correlation	ì .	intercept	-0.000	-0.000
		slope	0.766	0.798
		prob. slope = 0	0.000	0.000
		prob. slope = 1	0.028	0.074
	2			0.8900
				-0.000
				0.960
				0.000
				0.751
Spearman	1	Spearman's r	0.611	0.600
correlation	ì .	prob. r = 0	0.000	0.000
				0 500
	2			0.729
				0.000

Plots of the mean relative difference, for profile water content on the day after irrigation, showed that this variable was also quite time invariant (Figure 8-2). Linear and Spearman rank correlations were highly significant (Table 8-2). Slopes were close to 1, indicating that these data were time invariant by the criterion of Equation 8-4.

Table 8-2.

Linear correlations and Spearman rank-order correlations among profile water contents on the day after irrigation. All irrigations, Experiment 2. For linear correlations, from top to bottom, the numbers are the correlation coefficient, intercept, slope, probability that slope equals 0 (data sets uncorrelated) and probability that slope equals 1. For Spearman correlations the numbers are Spearman's coefficient, r, and the probability that ranks are not preserved.

	Irrigat:	lon\Irrigation	2	3
Simple	1	r	0.9755	0.9512
correlation	L	intercept	0.000	-0.003
		slope	1.015	0.930
		prob. slope = 0	0.000	0.000
		prob. slope = 1	0.888	0.503
	2			0.9777
				-0.000
				0.922
				0.000
				0.434
Spearman	1	Spearman's r	0.972	0.961
correlation	1	prob. r = 0	0.000	0.000
		_		
	2			0.990
				0.000
		•		

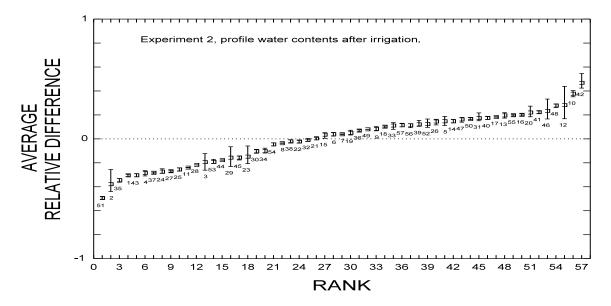


Figure 8-2. Average relative difference for all irrigations of profile water content on the day after irrigation [squares], ranked and with bars showing maximum and minimum values, Experiment 2.

In contrast, the change in storage due to irrigation was not significantly correlated between irrigations 1 and 2, nor between irrigations 1 and 3 (Table 8-2). The correlation for change in storage between irrigations 2 and 3 was significant at the 10% level but the correlation coefficient, at 0.25, was much lower than those for catch can depths and profile water content, both of which were greater than 0.8 for all comparisons. Slopes were never close to 1. A plot of the relative difference showed a high degree of dispersion for the change in storage, especially at the high end (Figure 8-3). This dispersion may be due to cracks that were open prior to the first irrigation. After the first irrigation the soil never dried enough for large cracks to open. The change in storage due to irrigation does

not seem to be time invariant.

Table 8-3.

Linear correlations and Spearman rank-order correlations among data sets for change in storage (cm) due to irrigation, for the 3 irrigations, Experiment 2. For linear correalations, from top to bottom, the numbers are correlation coefficient, intercept, slope, probability that the data sets are uncorrelated (slope = 0) and probability that the slope equals 1. For Spearman correlations the numbers are Spearman's coefficient, r, and probability that ranks are not preserved.

Simple Statistics

<u>Irrigation</u>	n N	Mean	Std Dev	Median	Minimum	<u> Maximum</u>
ĺ	57	3.8512	2.3182	3.4287	-0.4747	11.8260
2	57	0.8920	0.8507	0.7991	-1.5028	5.0642
3	55	0.7454	1.1025	0.5105	-0.1961	6.8423

	Irrigat:	ion\Irrigation	2	3
Simple	1	r	0.0997	-0.0985
correlation	1	intercept	0.000	-0.003
		slope	0.063	-0.038
		prob. slope = 0	0.461	0.475
		prob. slope = 1	0.000	0.000
	2			0.2548
				0.002
				0.165
				0.060
				0.000
Spearman	1	Spearman's r	0.304	-0.002
correlation	1	prob. r = 0	0.022	0.990
	2			0.418
				0.002

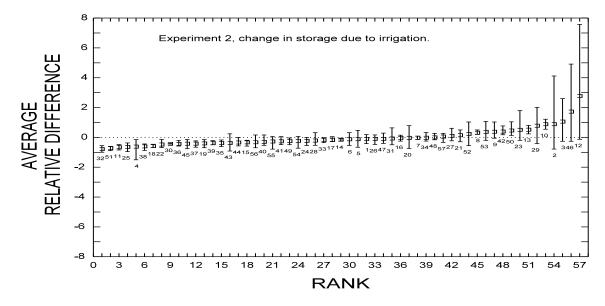


Figure 8-3. Average relative difference for all irrigations of change in storage due to irrigation [squares], ranked and with bars showing maximum and minimum values, Experiment 2.

Profile Water Contents.

Due to the large amount of space taken by the tables of linear and Spearman correlations, the remaining tables for this chapter are given in Appendix H. The daily distribution of profile water contents was also time invariant as the soil dried after irrigation (Table 8-4 [App. H], Figure 8-4). Correlation coefficients (r) were 0.97 or better no matter which two days after a given irrigation were compared. For instance the coefficients, for data from the first day after irrigation correlated against that from the last day before the next irrigation, were 0.98 for both Irrigations 1 and 2. Profile water contents for days from separate irrigations were almost as well correlated with the lowest coefficient being

0.94.

Spearman rank order correlation coefficients were only slightly lower than the linear correlations (Table 8-5 [App. H]). The probability that ranks were not preserved between any two days was 0.0001 indicating a very high degree of time invariance for the ranking of profile water content in this data set. This result was confirmed by the fact that, for 88 of 136 linear correlations, the slope was not significantly different from 1 at the 10 % level of probability. Thus Equation 8-4 is confirmed for these data.

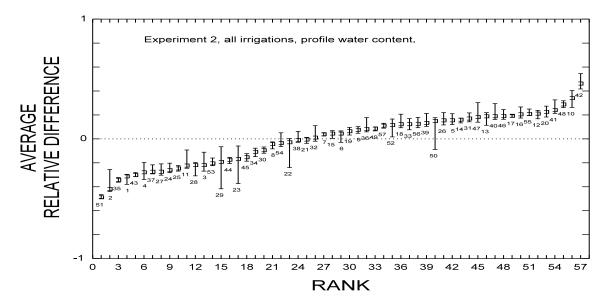


Figure 8-4. Average relative difference for all days of daily profile water content [squares], ranked and with bars showing maximum and minimum values, all irrigations, Experiment 2.

Surface Temperature.

Linear correlations on the midday soil temperature depression ($T_{o,max}$ - $T_{d,max}$) from different days at the 57 field locations showed that correlations were generally significant at the 10 % level but correlation coefficients were lower than those obtained for profile water content at the same locations (Table 8-6 [App. H]). Cloudiness on days 80 and 84 caused low or negative correlations between data for those days and all others. Data from days after Irrigation 1 did not correlate well with data from days after Irrigation 2. Otherwise data from days within one irrigation period generally was well correlated, with significance levels of 0.01% or better. Days after Irrigation 2 were especially well correlated, reflecting the generally less cloudy conditions occurring then. However the slope was significantly different from 1 (10 % level) for all but 2 of the correlations, showing that these data were not time invariant by the criterion of Equation 8-4.

Spearman rank correlation coefficients computed for the same surface temperature data behaved similarly to the Pearson correlations, indicating that as long as clear skies prevailed the ranking of soil surface temperatures in the field was relatively time invariant (Table 8-7 [App. H]).

Plotting the relative differences showed a high degree of dispersion for this data set (Figure 8-5) even with cloudy

days 80 and 84 omitted. Maxima and minima of the relative difference did not include zero for only 8 of the 57 locations.

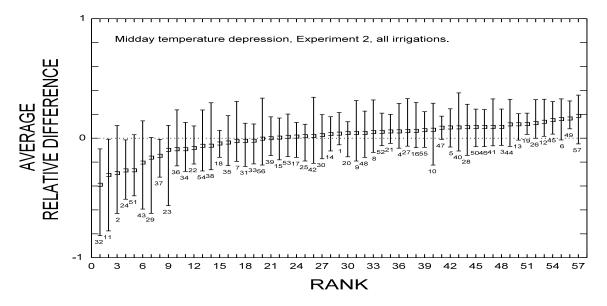


Figure 8-5. Average relative difference for $(T_{o,max} - T_{d,max})$ [squares], ranked and with bars showing maximum and minimum values, all irrigations, Experiment 2. Days 80 and 84 omitted.

Separate plots of the relative differences for Irrigations 1 and 2 showed that the more sunny conditions for Irrigation 2 resulted in a decrease in dispersion and in more stable ranking (Figure 8-6). These data were the average of 50 readings taken at 1 s intervals as the IR thermometer was rotated in a circular pattern to scan the area around each access tube. As discussed in Chapter 3 the variance of individual readings tended to be higher than the variance of temperature across the field. It should be interesting to compare these results with those obtained in Experiment 3 when IR temperatures were obtained from

spot readings of the ML surfaces. These data were time invariant by the ranking criterion but not by the stricter criterion of Equation 8-4.

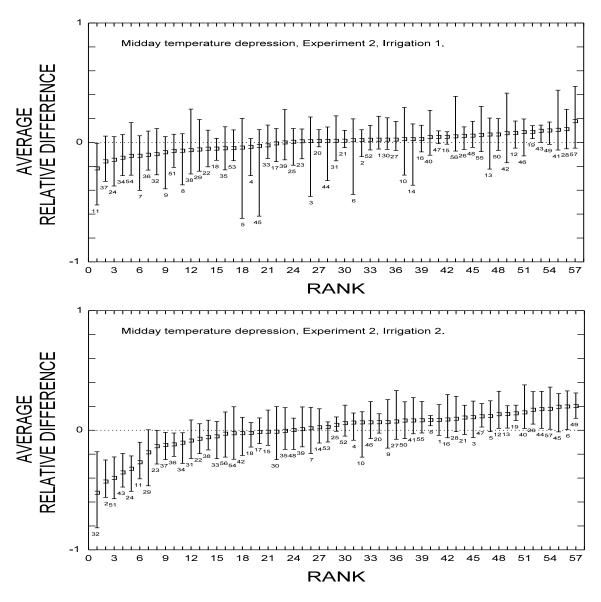


Figure 8-6. Average relative difference for $(T_{o,max} - T_{d,max})$ [squares], ranked and with bars showing maximum and minimum values. Experiment 2, top - Irrigation 1, bottom - Irrigation 2.

Experiment 3 Results.

Surface Temperature.

Spearman rank order correlations on midday ML surface temperatures $(T_{o,\text{max}} - T_{d,\text{max}})$ are shown in Table 8-9 (App. H). These data were obtained by pointing the infrared thermometer directly at the soil surface in the ML with no movement of the thermometer. This procedure was in contrast to that used in Experiment 2 when the infrared thermometer was moved in a circular pattern to scan the soil surface around each access tube site. For Irrigation 1 in Experiment 3 the 57 locations were identical to those used for access tube placement in Experiment 2. For Irrigation 2 all locations were displaced 1.5 m east.

Despite the differences in measurement method the Spearman correlations on Experiment 3 data showed the same general pattern (Table 8-9 [App. H]) as was just seen for Experiment 2 surface temperature data. For the days after each irrigation rank correlations were usually significant at the 0.001 level. Notable exceptions were for days 311 and 317 which were overcast and produced some negative correlations. Also day 329, which was the day after Irrigation 2, was not well correlated with subsequent days. As discussed later this may be an artifact of the extraction procedure for ML's on the first day after irrigation. Ranks were not well preserved across irrigations, with many low correlations between days

after Irrigation 1 and days after Irrigation 2. Rank preservation might not be expected in this case since the sampling locations were moved. It is interesting to note that movement of only 1.5 m was associated with the loss of correlation, indicating that the data were not highly autocorrelated even at this small distance.

Linear correlations showed the same pattern (Table 8-8 [App. H]). Although correlations were usually highly significant for days after either Irrigation 1 or 2, they were often not significant for days from different irrigations. Slopes ranged from 0.01 to 4.61. Only a few slopes were shown equal to 1 at the 10 % level. As for Experiment 2 temperature data, these data meet the ranking criterion for time invariance but not the criterion of Equation 8-4. Plots of the relative difference showed a high degree of dispersion for data from both Irrigations but much more dispersion for Irrigation 1 data (Figure 8-7).

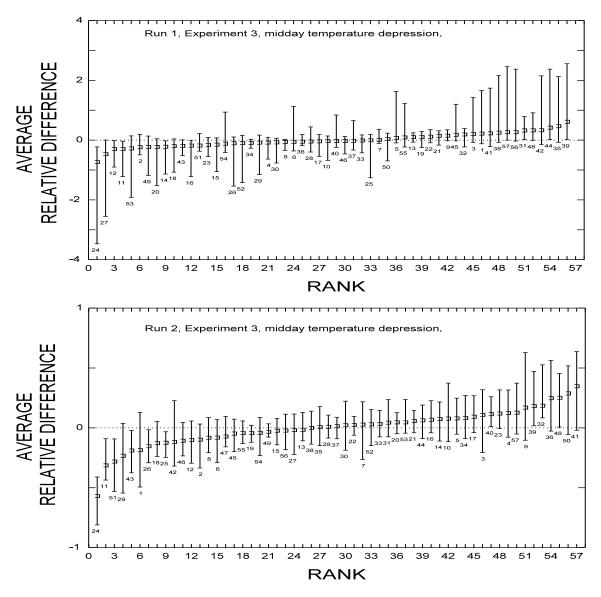


Figure 8-7. Average relative difference for $(T_{o,max} - T_{d,max})$ [squares], ranked and with bars showing maximum and minimum values. Experiment 3, top - Run 1, bottom - Run 2.

Evaporation.

Daily evaporation for Experiment 3 was measured at the 57 field locations using microlysimeters. Spearman correlations of daily ML evaporation showed that ranks were well preserved after each irrigation, usually to the 0.0001 significance level (Table 8-11 [App. H]). But ranks were not at all preserved between irrigations. These data were even more clear cut than the temperature data. Also there was not good correlation between data for the first day after Irrigation 2 (day 329) and subsequent days. Day 329 was unusually cold and the ML's were extracted and weighed before dawn. They then were left standing on the soil surface until the last ML was weighed. Only then were the ML's returned to plastic lined holes. It may be that the ML's were chilled during the intervening time with those extracted first being exposed the longest and chilled the most. Such treatment could have skewed the evaporation estimates for day 329. This may explain the poor correlation between the temperatures and evaporation measured on day 329 and those on subsequent days.

Linear correlations also showed data from a given post irrigation period to be well correlated with significance levels usually better than 0.0001% (Table 8-10 [App. H]). Correlations between irrigations were often not significant at the 10% level. Slopes were quite variable and generally were not equal to 1 at the 10% level. Plots of the relative

differences showed a high degree of dispersion in both data sets (Figure 8-8).

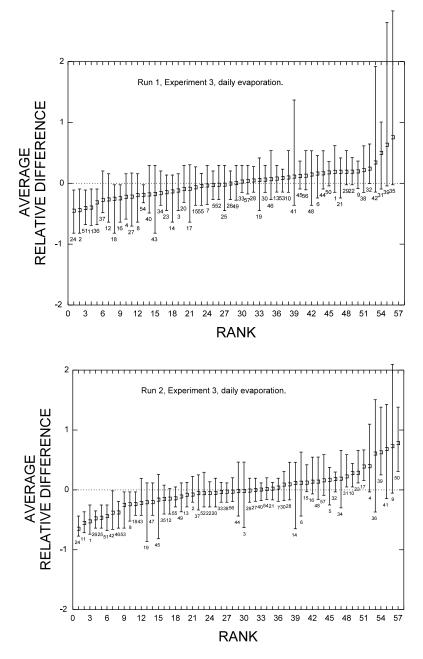


Figure 8-8. Average relative difference for daily evaporation from ML's [squares], ranked and with bars showing maximum and minimum values, Experiment 3, top - Run 1, bottom - Run 2.

Discussion.

The only variable shown to be unequivocally time invariant was the profile water content. The profile water content data presented here were more clearly time invariant than those presented by Kachanoski and De Jong (1988) and Ottoni (1984). This may be related to the facts that 1) the soil was bare (it supported vegetation in the 2 studies cited); and 2) the soil was fine textured in the surface and apparently had a low drainage rate. Textures ranged from "moderately fine textured" (Kachanoski and De Jong 1988) to a sandy loam (Ottoni 1984) in the studies cited. Van Wesenbeck et al. (1988) have shown that vegetation (in their case a corn crop) can selectively remove water from different areas of the soil resulting in a decrease in the temporal persistance of spatial patterns of soil water content.

Data for profile water contents of a silty clay, presented by Vachaud et al. (1985), appear to be similar to those presented here. Contrary to the report of Kachanoski and De Jong (1988), the linear correlation of relative differences for profile water contents had a slope that was close to 1, often significantly so, showing that these data were time invariant according to the criterion of Equation 8-4 proposed by those authors.

The lack of correlation for the change in storage due to irrigation is a similar result to the lack of correlation for recharge reported by Kachanoski and De Jong (1988).

Since profile water contents were very well correlated across irrigations for Experiment 2, while midday surface temperatures were not, then profile wetness is probably not a good indicator of surface temperature nor of evaporation which is well correlated with midday surface temperature. This does not mean that surface water content is not well correlated with either temperature or evaporation since no data were presented on surface water content.

The lack of rank correlation across irrigations, for the temperature and evaporation data of Experiment 3, indicates that the range of autocorrelation for these variables was smaller than 1.5 m. In the next chapter the autocorrelation range will be investigated more thoroughly. Although evaporation was well correlated (both Spearman and linear correlations) for days after a given irrigation, there was enough dispersion in the data to render problematic the picking of a site representative of the mean. The same was true for the surface temperature data. Thus the time invariance of surface temperature and evaporation were not clearly established.